SOFIA: $\mathcal{MQ}$-based signatures in the QROM

Ming-Shing Chen[1], Andreas Hülsing[2], **Joost Rijneveld**[3], Simona Samardjiska[3,4], and Peter Schwabe[3]

[1] National Taiwan University / Academia Sinica, Taipei, Taiwan
[2] Technische Universiteit Eindhoven, Eindhoven, The Netherlands
[3] Radboud University, Nijmegen, The Netherlands
[4] "Ss. Cyril and Methodius" University, Skopje, R. Macedonia

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Tenerife

$\mathcal{MQ}$-based signatures

- Important candidate for post-quantum signatures
- Several submissions to NIST
  - DualModeMS [FPR17], GeMSS [CFMR+17], Gui [PCY+15, DCP+17a], HiMQ-3 [SPK17], LUOV [BPSV17], MQDSS [CHR+16, CHR+17], Rainbow [DS05, DCP+17b]
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  - MQDSS: (lossy) ROM reduction to $\mathcal{MQ}$

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- SOFIA: continue in line of MQDSS
  - Transform an $\mathcal{MQ}$-based IDS

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- Lots of ongoing work!
- [KLP17]: tight Fiat-Shamir in the ROM
  - But similar issues in the QROM
- [KLS17]: Fiat-Shamir in QROM
  - Requires changing the IDS and parameters

This work

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4. Parameterize to achieve 128-bit post-quantum
   - SOFIA-4-128
5. Implement and compare on AVX2

Canonical Identification Schemes

$\mathcal{P}$		$\mathcal{V}$
$\mathsf{com} \leftarrow_R \mathcal{P}_0(\mathsf{sk})$	$\xrightarrow{\mathsf{com}}$	
	$\xleftarrow{\mathsf{ch}}$	$\mathsf{ch} \leftarrow_R \mathsf{ChS}(1^k)$
$\mathsf{resp} \leftarrow \mathcal{P}_1(\mathsf{sk}, \mathsf{com}, \mathsf{ch})$	$\xrightarrow{\mathsf{resp}}$	
		$b \leftarrow \mathsf{Vf}(\mathsf{pk}, \mathsf{com}, \mathsf{ch}, \mathsf{resp})$

Informally:

1. Prover commits to some (randomized) value derived from sk
2. Verifier picks a challenge ‘ch’
3. Prover computes response ‘resp’
4. Verifier checks if response matches challenge

Security of the IDS

- Passively secure IDS

Soundness: the probability that an adversary can convince is 'small'

Honest-Verifier Zero-Knowledge: simulator can 'fake' transcripts

Special soundness: two 'similar' transcripts $\Rightarrow$ secret exposed

Security of the IDS

- Passively secure IDS

Soundness: the probability that an adversary can convince is 'small'

- Adversary $\mathcal{A}$ can 'guess right': soundness error κ

$$\Pr\left[\begin{array}{l}(\mathsf{pk},\mathsf{sk}) \leftarrow \mathsf{KGen}(1^k)\\ \left\langle \mathcal{A}(1^k,\mathsf{pk}), \mathcal{V}(\mathsf{pk})\right\rangle = 1\end{array}\right] \leq \kappa + \mathsf{negl}(k).$$

Honest-Verifier Zero-Knowledge: simulator can 'fake' transcripts

- Shows transcripts do not leak the secret

Special soundness: two 'similar' transcripts $\Rightarrow$ secret exposed

- Proof relies on constructing an 'extractor'

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  - Adversary must have known several transcripts
  - Unblinding makes them available to extractor

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- Parallelize r rounds to decrease error
- Extra parameter: prepare for t challenges

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5-pass q2 Identification Schemes

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	$\xleftarrow{\alpha}$	$\alpha \leftarrow_R \mathbb{F}_q$
$\mathsf{resp}_1 \leftarrow \mathcal{P}_1(\mathsf{sk}, \mathsf{com}, \alpha)$	$\xrightarrow{\mathsf{resp}_1}$	
	$\xleftarrow{\mathsf{ch}_2}$	$\mathsf{ch}_2 \leftarrow_R \{0, 1\}$
$\mathsf{resp}_2 \leftarrow \mathcal{P}_2(\mathsf{sk}, \mathsf{com}, \alpha, \mathsf{resp}_1, \mathsf{ch}_2)$	$\xrightarrow{\mathsf{resp}_2}$	
		$b \leftarrow \mathsf{Vf}(\mathsf{pk}, \mathsf{com}, \alpha, \mathsf{resp}_1, \mathsf{ch}_2, \mathsf{resp}_2)$

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- Unruh's transform: resp_2 for both $\mathsf{ch}_2 \in \{0, 1\}$, per α

$\mathcal{MQ}$ problem

The function family $\mathcal{MQ}(n, m, \mathbb{F}_q)$:

$$\mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_m(\mathbf{x})), \text{ where } f_s(\mathbf{x}) = \sum_{i,j} a_{i,j}^{(s)} x_i x_j + \sum_i b_i^{(s)} x_i$$
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i.e., solve the system of equations:

$$\begin{aligned} y_1 &= a_{1,1}^{(1)} x_1 x_1 + a_{1,2}^{(1)} x_1 x_2 + \ldots + a_{n,n}^{(1)} x_n x_n + b_1^{(1)} x_1 + \ldots + b_n^{(1)} x_n \\ &\vdots \\ y_m &= a_{1,1}^{(m)} x_1 x_1 + a_{1,2}^{(m)} x_1 x_2 + \ldots + a_{n,n}^{(m)} x_n x_n + b_1^{(m)} x_1 + \ldots + b_n^{(m)} x_n \end{aligned}$$

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$$y_1 = 4x_1x_1 + 3x_1x_2 + 0x_1x_3 + x_2x_2 + 2x_2x_3 + x_3x_3 + 0x_1 + 2x_2 + 2x_3$$
$$y_2 = x_1x_1 + 2x_1x_2 + x_1x_3 + 0x_2x_2 + 3x_2x_3 + 4x_3x_3 + 0x_1 + 3x_2 + 2x_3$$
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- 'Secret' input $\mathbf{x} = (1, 4, 3)$

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$$y_1 = 4 \cdot 1 \cdot 1 + 3 \cdot 1 \cdot 4 + 4 \cdot 4 + 2 \cdot 4 \cdot 3 + 3 \cdot 3 + 2 \cdot 4 + 2 \cdot 3$$
$$y_2 = 1 \cdot 1 + 2 \cdot 1 \cdot 4 + 1 \cdot 3 + 3 \cdot 4 \cdot 3 + 4 \cdot 3 \cdot 3 + 3 \cdot 4 + 2 \cdot 3$$
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$$y_1 = 4 \cdot 1 \cdot 1 + 3 \cdot 1 \cdot 4 + 4 \cdot 4 + 2 \cdot 4 \cdot 3 + 3 \cdot 3 + 2 \cdot 4 + 2 \cdot 3 = 79 \equiv 4$$
$$y_2 = 1 \cdot 1 + 2 \cdot 1 \cdot 4 + 1 \cdot 3 + 3 \cdot 4 \cdot 3 + 4 \cdot 3 \cdot 3 + 3 \cdot 4 + 2 \cdot 3 = 102 \equiv 2$$
$$y_3 = 1 \cdot 4 + 4 \cdot 1 \cdot 3 + 3 \cdot 4 \cdot 4 + 3 \cdot 3 + 4 \cdot 1 + 4 = 81 \equiv 1$$

- 'Public' output $\mathbf{y} = (4, 2, 1)$

Sakumoto-Shirai-Hiwatari 5-pass IDS [SSH11]

$\mathcal{P} : (\mathbf{F}, \mathbf{v}, \mathbf{s})$		$\mathcal{V} : (\mathbf{F}, \mathbf{v})$
$\mathbf{r}_0, \mathbf{t}_0 \leftarrow_R \mathbb{F}_q^n, \mathbf{e}_0 \leftarrow_R \mathbb{F}_q^m$		
$\mathbf{r}_1 \leftarrow \mathbf{s} - \mathbf{r}_0$		
$c_0 \leftarrow Com(\mathbf{r}_0, \mathbf{t}_0, \mathbf{e}_0)$		
$c_1 \leftarrow Com(\mathbf{r}_1, \mathbf{G}(\mathbf{t}_0, \mathbf{r}_1) + \mathbf{e}_0)$	$\xrightarrow{(c_0, c_1)}$	
	$\xleftarrow{\alpha}$	$\alpha \leftarrow_R \mathbb{F}_q$
$\mathbf{t}_1 \leftarrow \alpha \mathbf{r}_0 - \mathbf{t}_0$		
$\mathbf{e}_1 \leftarrow \alpha \mathbf{F}(\mathbf{r}_0) - \mathbf{e}_0$	$\xrightarrow{\text{resp}_1 = (\mathbf{t}_1, \mathbf{e}_1)}$	
	$\xleftarrow{\text{ch}_2}$	$\text{ch}_2 \leftarrow_R \{0, 1\}$
If $\text{ch}_2 = 0$, $\text{resp}_2 \leftarrow \mathbf{r}_0$		
Else $\text{resp}_2 \leftarrow \mathbf{r}_1$	$\xrightarrow{\text{resp}_2}$	
		If $\text{ch}_2 = 0$, Parse $\text{resp}_2 = \mathbf{r}_0$, check
		$c_0 \stackrel{?}{=} Com(\mathbf{r}_0, \alpha \mathbf{r}_0 - \mathbf{t}_1, \alpha \mathbf{F}(\mathbf{r}_0) - \mathbf{e}_1)$
		Else Parse $\text{resp}_2 = \mathbf{r}_1$, check
		$c_1 \stackrel{?}{=} Com(\mathbf{r}_1, \alpha(\mathbf{v} - \mathbf{F}(\mathbf{r}_1)) - \mathbf{G}(\mathbf{t}_1, \mathbf{r}_1) - \mathbf{e}_1)$

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$\mathcal{P} : (\mathbf{F}, \mathbf{v}, \mathbf{s})$ — $\mathcal{V} : (\mathbf{F}, \mathbf{v})$

Prover:

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$\mathbf{r}_1 \leftarrow \mathbf{s} - \mathbf{r}_0$

$c_0 \leftarrow Com(\mathbf{r}_0, \mathbf{t}_0, \mathbf{e}_0)$

$c_1 \leftarrow Com(\mathbf{r}_1, \mathbf{G}(\mathbf{t}_0, \mathbf{r}_1) + \mathbf{e}_0)$

$\xrightarrow{(c_0, c_1)}$

Verifier: $\alpha \leftarrow_R \mathbb{F}_q$

$\xleftarrow{\alpha}$

Prover:

$\mathbf{t}_1 \leftarrow \alpha \mathbf{r}_0 - \mathbf{t}_0$

$\mathbf{e}_1 \leftarrow \alpha \mathbf{F}(\mathbf{r}_0) - \mathbf{e}_0$

$\xrightarrow{\mathsf{resp}_1 = (\mathbf{t}_1, \mathbf{e}_1)}$

Verifier: $\mathsf{ch}_2 \leftarrow_R \{0, 1\}$

$\xleftarrow{\mathsf{ch}_2}$

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Sakumoto-Shirai-Hiwatari IDS [SSH11]

- Key technique: cut-and-choose for $\mathcal{MQ}$
  - Analogously, consider DLP: $s = r_0 + r_1 \Rightarrow g^s = g^{r_0} \cdot g^{r_1}$

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- Bilinear map $\mathbf{G}(\mathbf{x}, \mathbf{y}) = \mathbf{F}(\mathbf{x} + \mathbf{y}) - \mathbf{F}(\mathbf{x}) - \mathbf{F}(\mathbf{y})$
  - Split $\mathbf{s}$ and $\mathbf{F}(\mathbf{s})$ into $\mathbf{r}_0, \mathbf{r}_1$ and $\mathbf{F}(\mathbf{r}_0), \mathbf{F}(\mathbf{r}_1)$
    - Since then $\mathbf{s} = \mathbf{r}_0 + \mathbf{r}_1 \Rightarrow \mathbf{F}(\mathbf{s}) = \mathbf{G}(\mathbf{r}_0, \mathbf{r}_1) + \mathbf{F}(\mathbf{r}_0) + \mathbf{F}(\mathbf{r}_1)$
  - Split $\mathbf{r}_0$ and $\mathbf{F}(\mathbf{r}_0)$ further into $\mathbf{t}_0, \mathbf{t}_1$ resp. $\mathbf{e}_0, \mathbf{e}_1$

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  - For $g_s \in \mathbf{G}$: $g_s(\mathbf{x}, \mathbf{y}) = \sum_{i,j} a_{i,j}^{(s)}(x_i y_j + x_j y_i)$
    - Recall: $f_s(\mathbf{x}) = \sum_{i,j} a_{i,j}^{(s)} x_i x_j + \sum_i b_i^{(s)} x_i$

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  - See [SSH11] for details
  - Takeaway: evaluating $\mathbf{G} \approx$ evaluating $\mathbf{F}$

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    - Since then $\mathbf{s} = \mathbf{r}_0 + \mathbf{r}_1 \Rightarrow \mathbf{F}(\mathbf{s}) = \mathbf{G}(\mathbf{r}_0, \mathbf{r}_1) + \mathbf{F}(\mathbf{r}_0) + \mathbf{F}(\mathbf{r}_1)$
  - Split $\mathbf{r}_0$ and $\mathbf{F}(\mathbf{r}_0)$ further into $\mathbf{t}_0, \mathbf{t}_1$ resp. $\mathbf{e}_0, \mathbf{e}_1$
  - For $g_s \in \mathbf{G}$: $g_s(\mathbf{x}, \mathbf{y}) = \sum_{i,j} a_{i,j}^{(s)}(x_i y_j + x_j y_i)$
    - Recall: $f_s(\mathbf{x}) = \sum_{i,j} a_{i,j}^{(s)} x_i x_j + \sum_i b_i^{(s)} x_i$
  - See [SSH11] for details
  - Takeaway: evaluating $\mathbf{G} \approx$ evaluating $\mathbf{F}$
- Result: reveal either $\mathbf{r}_0$ or $\mathbf{r}_1$, and $(\mathbf{t}_0, \mathbf{e}_0)$ or $(\mathbf{t}_1, \mathbf{e}_1)$

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Key generation:

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Verification:

- Reconstruct indices
- Verify revealed responses
- Verify that commitments match responses; $r \times \mathbf{F}$, $\backsim \frac{1}{2} r \times \mathbf{G}$

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Many similarities to e.g. Picnic [CDG+17]

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What doesn't help:

- Opening for multiple α
- Committing to multiple $\mathbf{t}_0$

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- $t = 3$, $r = 438$ (since $2^{-(r \log \frac{2t}{t+1})/2} < 2^{-128}$)
- XOFs, hashes, PRGs: SHAKE, cSHAKE, (AES)

Implementation

- Evaluating $\mathcal{MQ}$
- XOFs

Implementation

- Evaluating $\mathcal{MQ}$
  - 438 rounds, 2x per round
  - Pairwise multiply 128x $\in \mathbb{F}_4$
  - Multiply by coefficients from **F**, $\in \mathbb{F}_4$
  - Accumulate
- XOFs
  - Blinding commitments
  - Expanding **F**: 262 KiB
  - External parallelism and cSHAKE

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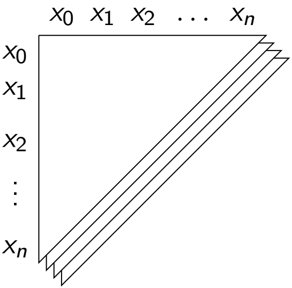

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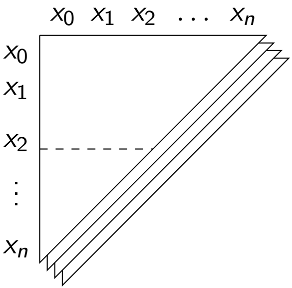

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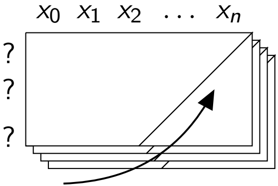

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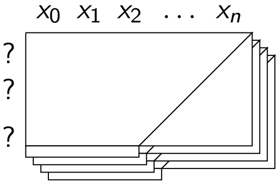

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$$\begin{aligned} c_{high} &= (a_{high} \wedge (b_{high} \oplus b_{low})) \oplus (a_{low} \wedge b_{high}) \\ c_{low} &= (a_{low} \wedge b_{low}) \oplus (a_{high} \wedge b_{high}) \end{aligned}$$

- `vpand`, `vpand`, `vpermq`, `vpxor`

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- Both cases: external parallelism over constant **F**

- Horizontal in batches of 3, avg. 17 558 cycles per $\mathcal{MQ}$

SOFIA-4-128 vs MQDSS-31-64

a.k.a. the price of QROM

- Signature size: 123 KiB (MQDSS: 40 KiB)
- 64 bytes pk, 32 bytes sk (MQDSS: 72 B, 64 B)

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- Key generation 1.16 M cycles (MQDSS: 1.18 M)
- Signing 21.31 M cycles (MQDSS: 8.51 M)
  - ∽75% $\mathcal{MQ}$
  - ∽25% SHAKE
- Verification 15.49 M cycles (MQDSS: 5.75 M)

(Intel Haswell, Core-i7-4770K, AVX2)

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- C and AVX2 code available (public domain): `https://joostrijneveld.nl/papers/sofia`

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