From 5-pass $\mathcal{MQ}$-based identification to $\mathcal{MQ}$-based signatures

Ming-Shing Chen[1,2], Andreas Hülsing[3], **Joost Rijneveld**[4], Simona Samardjiska[5], Peter Schwabe[4]

National Taiwan University[1] / Academia Sinica[2], Taipei, Taiwan
Eindhoven University of Technology, The Netherlands[3]
Radboud University, Nijmegen, The Netherlands[4]
"Ss. Cyril and Methodius" University, Skopje, Republic of Macedonia[5]

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Post-quantum signatures

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- Security arguments
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Solutions:

- Hash-based: SPHINCS [BHH+15], XMSS [BDH11, HRS16]
  - Slow or stateful
- Lattice-based: (Ring-)TESLA [ABB+16, ABB+15], BLISS [DDL+13], GLP [GLP12]
  - Large keys, or additional structure
- $\mathcal{MQ}$: ?
  - Unclear security: many broken (except HFEv-, UOV)

This work

- Transform class of 5-pass IDS to signature schemes
  - Extend Fiat Shamir transform
- Prove an earlier attempt [EDV+12] vacuous
  - Amended in [DGV+16]
- Propose MQDSS
  - Obtained by performing transform
  - Hardness of $\mathcal{MQ}$
- Instantiate and implement as MQDSS-31-64

But also:

- Reduction in the ROM (not in QROM)
- No tight proof

Canonical Identification Schemes

$\mathcal{P}$		$\mathcal{V}$
$\mathsf{com} \leftarrow \mathcal{P}_0(\mathsf{sk})$	$\xrightarrow{\mathsf{com}}$	
	$\xleftarrow{\mathsf{ch}}$	$\mathsf{ch} \leftarrow_R \mathsf{ChS}(1^k)$
$\mathsf{resp} \leftarrow \mathcal{P}_1(\mathsf{sk}, \mathsf{com}, \mathsf{ch})$	$\xrightarrow{\mathsf{resp}}$	
		$b \leftarrow \mathsf{Vf}(\mathsf{pk}, \mathsf{com}, \mathsf{ch}, \mathsf{resp})$

Informally:

1. Prover commits to some (random) value derived from sk
2. Verifier picks a challenge 'ch'
3. Prover computes response 'resp'
4. Verifier checks if response matches challenge

Security of the IDS

- Passively secure IDS

Soundness: the probability that an adversary can convince is 'small'

Honest-Verifier Zero-Knowledge: simulator can 'fake' transcripts

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Soundness: the probability that an adversary can convince is 'small'

- Shows knowledge of secret
- Adversary $\mathcal{A}$ can 'guess right': soundness error κ

$$\Pr\left[\begin{array}{l}(\mathsf{pk},\mathsf{sk}) \leftarrow \mathsf{KGen}(1^k) \\ \left\langle \mathcal{A}(1^k,\mathsf{pk}), \mathcal{V}(\mathsf{pk})\right\rangle = 1\end{array}\right] \leq \kappa + \mathsf{negl}(k).$$

Honest-Verifier Zero-Knowledge: simulator can 'fake' transcripts

- Shows that transcripts do not leak the secret

Fiat-Shamir transform

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  - Transcript is signature
- Generalize to 5-pass
  - Benefit from lower soundness error

5-pass Fiat-Shamir transform

- Attempt in [EDV+12] incorrect
  - *'n-soundness'*
    - Two transcripts agree up to last challenge $\Rightarrow$ extract sk
- Vacuous assumption: satisfying schemes reduce to 3-pass
  - HVZK: combine first 3 messages into 1
  - Special soundness: transform transcripts, use extractor

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- n-soundness fixed in [DGV+16]
  - Still does not apply to existing schemes

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- Restrict to challenge spaces of size q resp. 2
  - *'q2-IDS'*
- Prove EU-CMA using dedicated forking lemma
  - Assuming a successful forgery ..
  - .. generate 4 signatures fulfilling pattern on challenges
  - .. obtain 4 traces with same commitments, pattern on challenges
  - Use *q2-IDS* that allow extracting sk

$\mathcal{MQ}$ problem

The function family $\mathcal{MQ}(n, m, \mathbb{F}_q)$:

$$\mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_m(\mathbf{x})), \text{ where } f_s(\mathbf{x}) = \sum_{i,j} a_{i,j}^{(s)} x_i x_j + \sum_i b_i^{(s)} x_i$$
$$\text{for } a_{i,j}^{(s)},\ b_i^{(s)} \in \mathbb{F}_q, s \in \{1, \ldots, m\}$$

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Problem: For given $\mathbf{y} \in \mathbb{F}_q^m$, find $\mathbf{x} \in \mathbb{F}_q^n$ such that $\mathbf{F}(\mathbf{x}) = \mathbf{y}$.

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i.e., solve the system of equations:

$$\begin{aligned} y_1 &= a_{1,1}^{(1)} x_1 x_1 + a_{1,2}^{(1)} x_1 x_2 + \ldots + a_{n,n}^{(1)} x_n x_n + b_1^{(1)} x_1 + \ldots + b_n^{(1)} x_n \\ &\vdots \\ y_m &= a_{1,1}^{(m)} x_1 x_1 + a_{1,2}^{(m)} x_1 x_2 + \ldots + a_{n,n}^{(m)} x_n x_n + b_1^{(m)} x_1 + \ldots + b_n^{(m)} x_n \end{aligned}$$

Sakumoto-Shirai-Hiwatari 5-pass IDS [SSH11]

$\mathcal{P} : (\mathbf{F}, \mathbf{v}, \mathbf{s})$		$\mathcal{V} : (\mathbf{F}, \mathbf{v})$
$\mathbf{r}_0, \mathbf{t}_0 \leftarrow_R \mathbb{F}_q^n, \mathbf{e}_0 \leftarrow_R \mathbb{F}_q^m$		
$\mathbf{r}_1 \leftarrow \mathbf{s} - \mathbf{r}_0$		
$c_0 \leftarrow Com(\mathbf{r}_0, \mathbf{t}_0, \mathbf{e}_0)$		
$c_1 \leftarrow Com(\mathbf{r}_1, \mathbf{G}(\mathbf{t}_0, \mathbf{r}_1) + \mathbf{e}_0)$	$\xrightarrow{(c_0, c_1)}$	
	$\xleftarrow{\alpha}$	$\alpha \leftarrow_R \mathbb{F}_q$
$\mathbf{t}_1 \leftarrow \alpha\mathbf{r}_0 - \mathbf{t}_0$		
$\mathbf{e}_1 \leftarrow \alpha\mathbf{F}(\mathbf{r}_0) - \mathbf{e}_0$	$\xrightarrow{\mathsf{resp}_1 = (\mathbf{t}_1, \mathbf{e}_1)}$	
	$\xleftarrow{\mathsf{ch}_2}$	$\mathsf{ch}_2 \leftarrow_R \{0, 1\}$
If $\mathsf{ch}_2 = 0$, $\mathsf{resp}_2 \leftarrow \mathbf{r}_0$		
Else $\mathsf{resp}_2 \leftarrow \mathbf{r}_1$	$\xrightarrow{\mathsf{resp}_2}$	
		If $\mathsf{ch}_2 = 0$, Parse $\mathsf{resp}_2 = \mathbf{r}_0$, check $c_0 \stackrel{?}{=} Com(\mathbf{r}_0, \alpha\mathbf{r}_0 - \mathbf{t}_1, \alpha\mathbf{F}(\mathbf{r}_0) - \mathbf{e}_1)$
		Else Parse $\mathsf{resp}_2 = \mathbf{r}_1$, check $c_1 \stackrel{?}{=} Com(\mathbf{r}_1, \alpha(\mathbf{v} - \mathbf{F}(\mathbf{r}_1)) - \mathbf{G}(\mathbf{t}_1, \mathbf{r}_1) - \mathbf{e}_1)$

Sakumoto-Shirai-Hiwatari 5-pass IDS [SSH11]

- Relies only on $\mathcal{MQ}$, not IP
- Key technique: cut-and-choose for $\mathcal{MQ}$
  - Analogously, consider DLP: $s = r_0 + r_1 \Rightarrow g^s = g^{r_0} \cdot g^{r_1}$
- Bilinear map $\mathbf{G}(\mathbf{x}, \mathbf{y}) = \mathbf{F}(\mathbf{x} + \mathbf{y}) - \mathbf{F}(\mathbf{x}) - \mathbf{F}(\mathbf{y})$
  - Split $\mathbf{s}$ and $\mathbf{F}(\mathbf{s})$ into $\mathbf{r}_0, \mathbf{r}_1$ and $\mathbf{F}(\mathbf{r}_0), \mathbf{F}(\mathbf{r}_1)$
  - Split again into $\mathbf{t}_0, \mathbf{t}_1$ resp. $\mathbf{e}_0, \mathbf{e}_1$, using α
  - See [SSH11] for details
- Result: reveal either $(\mathbf{r}_0, \mathbf{t}_1, \mathbf{e}_1)$ or $(\mathbf{r}_1, \mathbf{t}_1, \mathbf{e}_1)$

MQDSS

- Generate keys
  - Sample seed $\mathcal{S}_F \in \{0,1\}^k$, $\mathbf{sk} \in \mathbb{F}_q^n$ $\Rightarrow (\mathcal{S}_F, \mathbf{sk})$
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    - $2r$ $\mathcal{MQ}$ evaluations

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  - Check combined commitments hash
- Parameters: k, n, m, $\mathbb{F}_q$, Com, hash functions, PRGs

MQDSS-31-64

- Security parameter $k = 256$ ($\Rightarrow$ 128-bit PQ security)
- Soundness error κ depends on q
  - $\kappa = \frac{q+1}{2q}$
  - Determines number of rounds: $r = 269$, $\kappa^{269} < (\frac{1}{2})^{256}$
- $\mathbb{F}_q = \mathbb{F}_{31}$, $n = m = 64$
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- Signature σ contains:
  - R, for random digest $\Rightarrow$ 32B
  - Hash $\mathcal{H}(commits)$ $\Rightarrow$ 32B
  - For every round: $\Rightarrow 269 \times$
    - Response vectors **t**, **e**, **r** $\Rightarrow 3 \times 40$B
    - 'Missing commit' $\Rightarrow$ 32B

Evaluating $\mathcal{MQ}$

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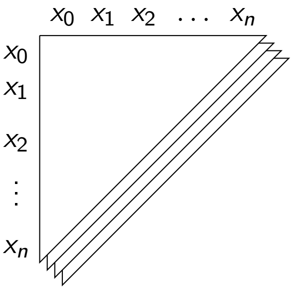

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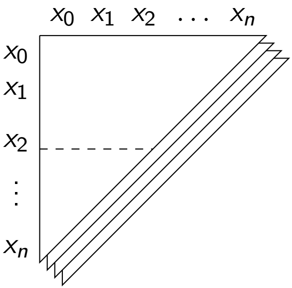

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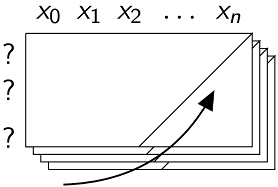

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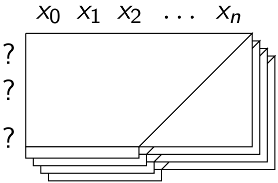

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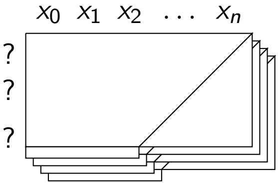

- Compute monomials, evaluate polynomials
- 64 elements in $\mathbb{F}_{31}$; 16 (or 32) per 256 bit AVX2 register

Benchmarks & conclusion

- Signatures: ~40 KB (≈ SPHINCS)
- Public and private keys: 72 resp. 64 bytes
- Signing time: ~8.5M cycles (2.43ms @ 3.5GHz)
  - Verification 5.2M, key generation 1.8M
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- Code is available (public domain):
  https://joostrijneveld.nl/papers/mqdss/

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