ARMed SPHINCS:

Computing a 41 KB signature in 16 KB of RAM

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The Netherlands

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DS lunch talk

Context

- SPHINCS[1]: Stateless, practical, **hash-based**, incredibly nice cryptographic signatures
- Hash functions do not fall to Shor (but halved by Grover)
- Hash-based schemes: conservative choice post-quantum
  - Necessary building block for signatures
  - Tight security reductions

[1] Daniel J. Bernstein, Diana Hopwood, Andreas Hülsing, Tanja Lange, Ruben Niederhagen, Louiza Papachristodoulou, Peter Schwabe and Zooko Wilcox O'Hearn, *Eurocrypt 2015*

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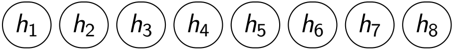

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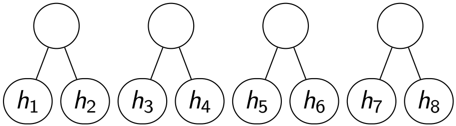

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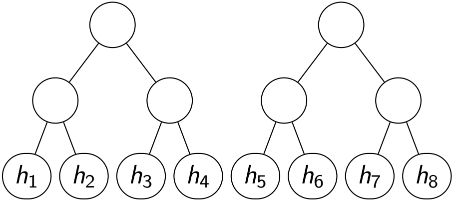

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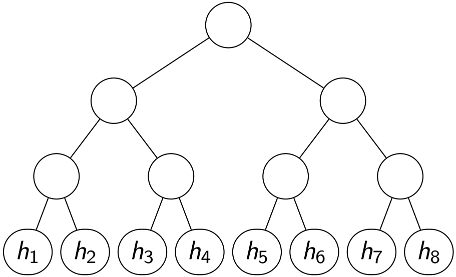

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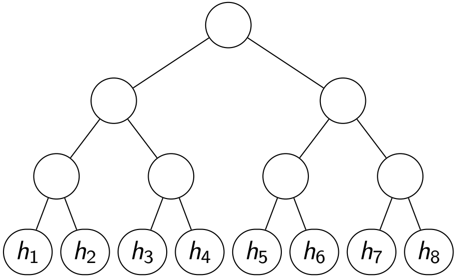

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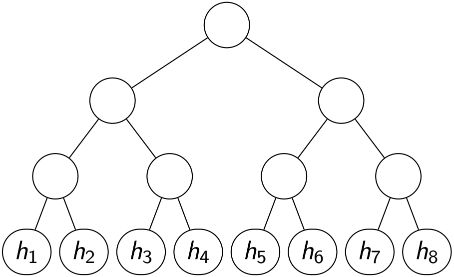
  - Position in the Merkle tree, e.g. 5
  - Nodes along the *authentication path*



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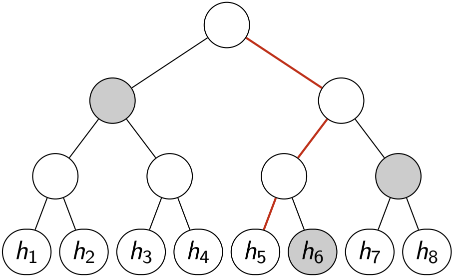

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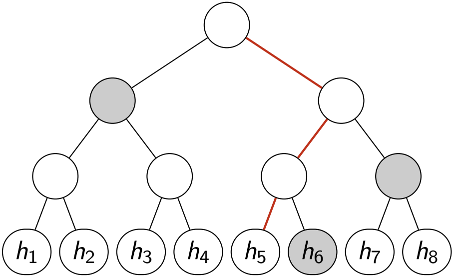

- Verification: verify σ, reconstruct root node

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- Signing is fast
- Keys are small
  - Private key generated from small seed
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- Signatures are somewhat large..
- Need to **remember** the last used index!
  - Terribly inconvenient

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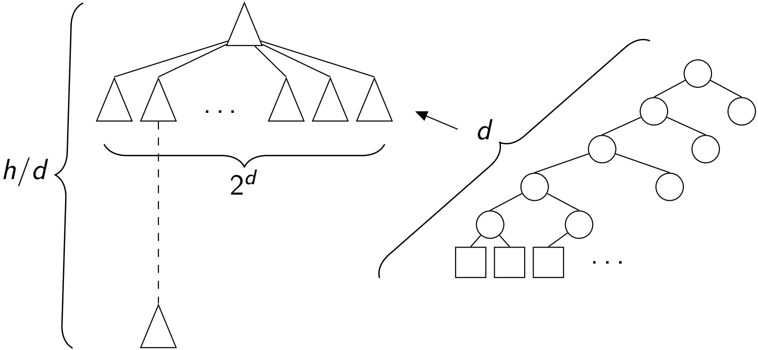
- Large Merkle tree, height h
- Every d-th layer signs child node using an OTS
  - Effectively a hypertree of h/d Merkle trees
- Sign messages using leaf nodes



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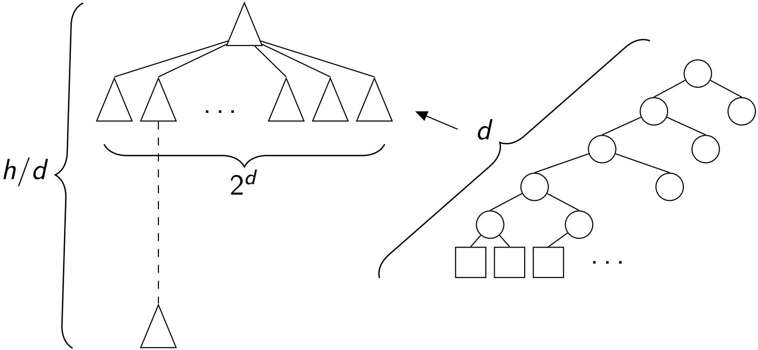

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- Layers of OTS: no need to compute entire tree
- Layers of hashing: acceptable signature size
- ‘Few time signature scheme’ (FTS) for leaf nodes
- Chance of a break becomes negligible

Key generation

- Generate random values SK_1 and SK_2
- Use SK_1: generate OTS keys of top sub-tree
- Compute root node (recall: each sub-tree is a Merkle tree)
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- In general: SK_1 generates OTS and FTS keys *deterministically*

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- Repeat.. until root node
- Signature: $\Sigma = (R, \sigma_{FTS}, (\sigma_{OTS_1}, Auth_1), (\sigma_{OTS_2}, Auth_2), \ldots, (\sigma_{OTS_{h/d}}, Auth_{h/d}))$

SPHINCS-256

- 41KB signatures, 1KB keys
- 256-bit hash functions
    - 128-bit post-quantum security
- $h = 60, d = 5$: 12 layers of sub-trees
- 2^{60} leaf nodes

Building blocks

- OTS
- Hash functions
- Key expansion function
- FTS

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- Hash functions: *BLAKE*, π_{ChaCha}
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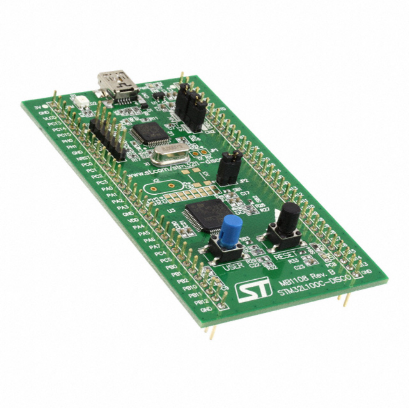
- FTS: *HORST*
  - Contains 16-layer Merkle tree (so $2^{16} = 65536$ leafs)
  - Goal: 32 authentication paths, root node
  - Paths start at (deterministically chosen) random leafs
  - Complete tree takes approx. 2MB RAM..

Platform

- STM32L100C board with Cortex M3
  - libopencm3 firmware
  - 32MHz, 32-bit architecture
  - 256KB Flash
  - **16KB RAM**



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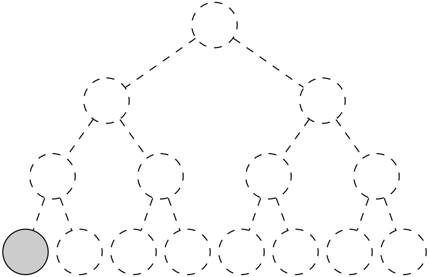

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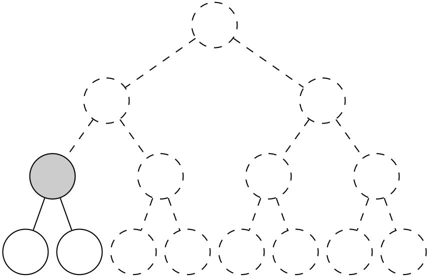

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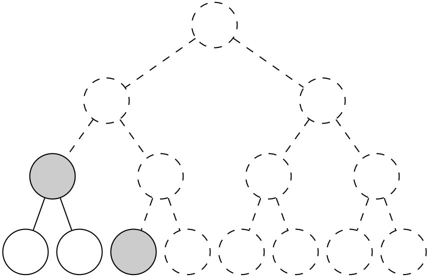

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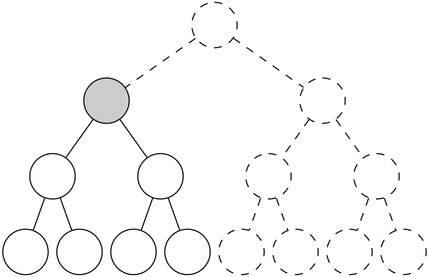

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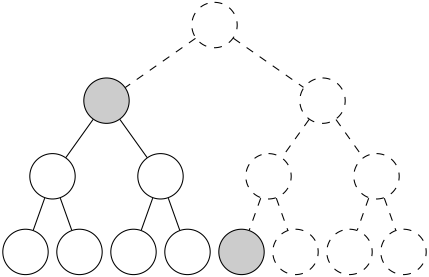

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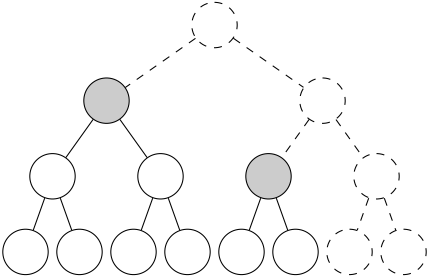

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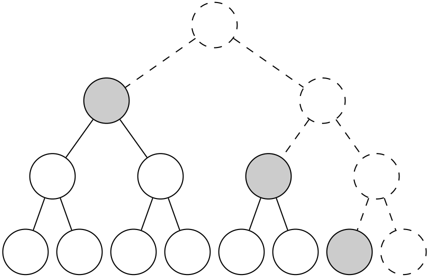

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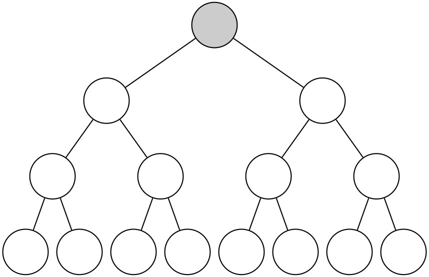

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- Output in the appropriate order..

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- Streaming message input
  - Blockwise BLAKE512
  - Stream twice: once for randomness, once for digest

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- ARMv7 rotates are (almost) free!
  - `eor r6, r6, r11, ROR #29`

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- On 4-core Haswell:

  “[..] signs hundreds of messages per second.”

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- Avg. signing: 19 441 021 cycles
- Verification: 4 961 447 cycles

Conclusions

- Stateless is expensive, but not prohibitively so
  - Signing 30x as expensive as XMSS^{MT}
  - Verification similar to XMSS^{MT}
  - (Key generation much cheaper)
- Feasible on limited platforms
  - Verification is practical
  - Non-interactive signatures (high latency)
- Further algorithmic improvements desirable

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- Public key: hash w times

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- Public key: hash w times, and construct an L-tree

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  - For this example, assume $m = 4$, so $w = 16$

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- Verification: complete hashes to w, check with public key

HORST

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- Very small chance of re-use