Implementing SPHINCS with restricted memory

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Master Thesis in CS
Radboud University

May 2015

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- This talk:
  - Relevant crypto context
  - SPHINCS
  - Implementation details
- Not this talk:
  - Background on public key crypto / hashes in general
  - Other post-quantum crypto
  - Quantum computing / crypto

Cryptographic context

- SPHINCS[1]: Stateless, practical, **hash-based**, incredibly nice cryptographic signatures
- Hashes do not fall to Shor (but halved by Grover)
- Hash-based schemes: conservative choice post-quantum
  - Fundamental building block

[1] Daniel J. Bernstein, Diana Hopwood, Andreas Hülsing, Tanja Lange, Ruben Niederhagen, Louiza Papachristodoulou, Peter Schwabe and Zooko Wilcox O'Hearn, 2015

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h_1 h_2 h_3 h_4 h_5 h_6 h_7 h_8

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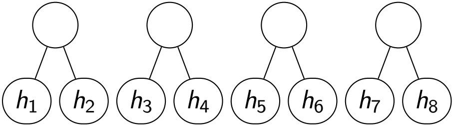

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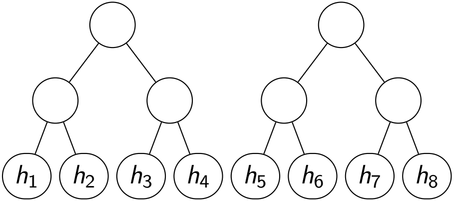

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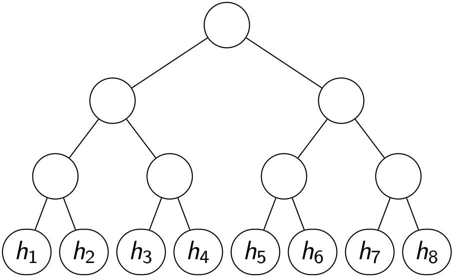

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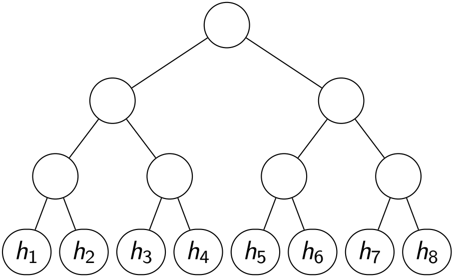

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- New public key: root node

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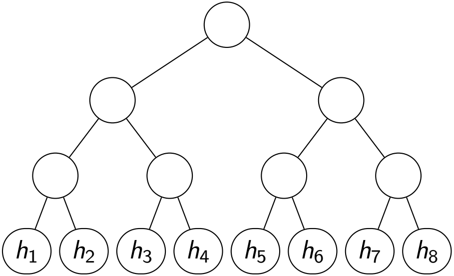
  - Position in the Merkle tree, e.g. 5
  - Nodes along the *authentication path*



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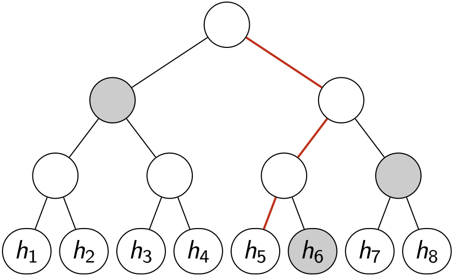

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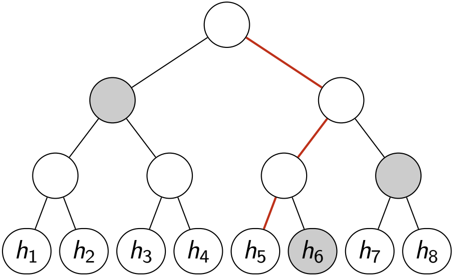

- Verification: reconstruct root node

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- Signing is fast
- Keys are small
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- Need to **remember** the last used index!
  - Terribly inconvenient

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- Every d-th layer signs child node using an OTS
  - Effectively a hypertree of h/d Merkle trees
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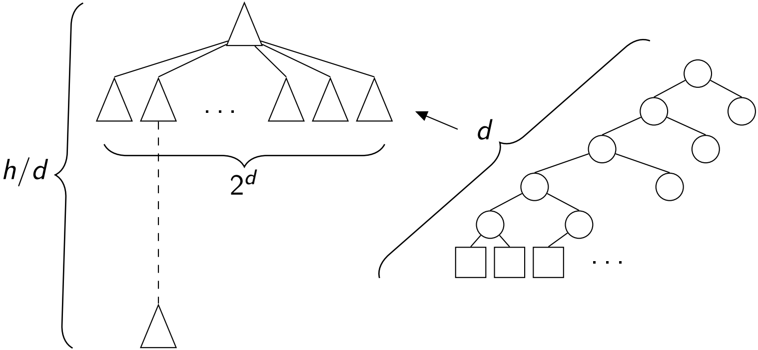

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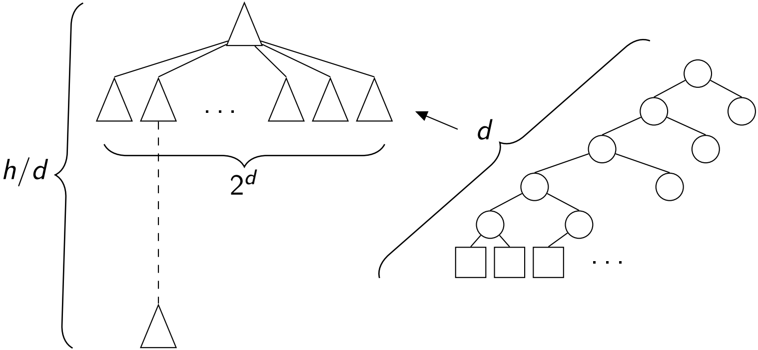

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- Layers of OTS: no need to compute entire tree
- Layers of hashing: acceptable signature size
- 'Few time signature scheme' (FTS) for leaf nodes
- Chance of a break becomes negligible

Key generation

- Generate random values SK_1 and SK_2
- Use SK_1: generate OTS keys of top sub-tree
- Compute root node (recall: the sub-tree is a Merkle tree)
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- In general: SK_1 generates OTS and FTS keys *deterministically*

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- Repeat.. until root node
- Signature: $\Sigma = (R, \sigma_{FTS}, (\sigma_{OTS_1}, Auth_1), (\sigma_{OTS_2}, Auth_2), \ldots, (\sigma_{OTS_{h/d}}, Auth_{h/d}))$

SPHINCS-256

- 41KB signatures, 1KB keys
- 256-bit hash functions
  - 128-bit post-quantum security
- $h = 60, d = 5$: 12 layers of sub-trees
- 2^{60} leaf nodes

Building blocks

- OTS
- Hash functions
- Key expansion function
- FTS

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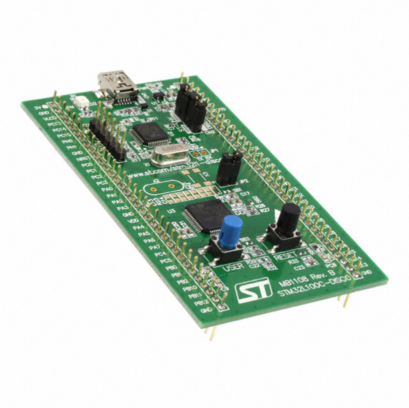
- FTS: *HORST*
  - Contains 16-layer Merkle tree (so 2^{16} leafs)
  - Goal: 32 authentication paths, root node
  - Complete tree takes approx. 2MB RAM..

Platform and implementation

- STM32L100C board with Cortex M3
  - libopencm3 firmware
  - 32MHz, 32-bit architecture
  - **16KB RAM**



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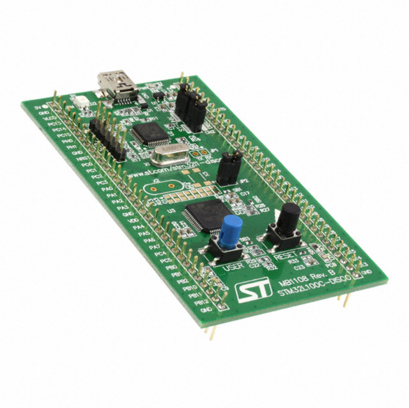
- Based on SPHINCS-256 for Haswell
  - Replaced `asm` with other implementations



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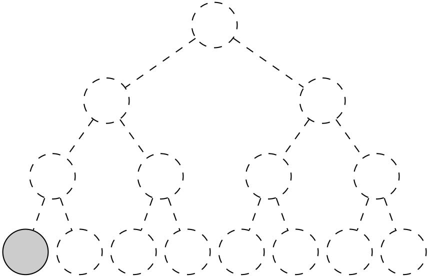

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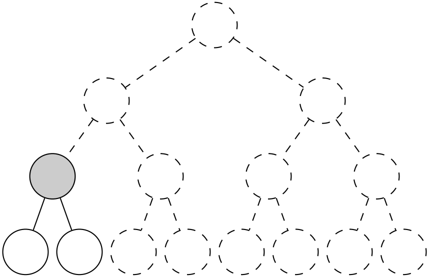

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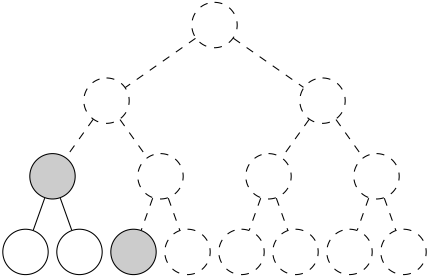

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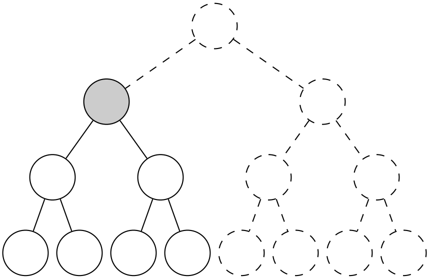

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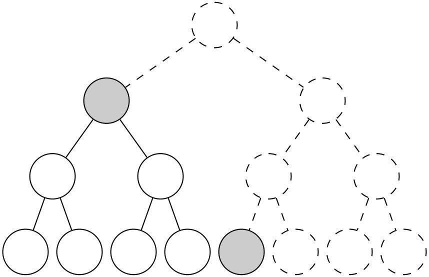

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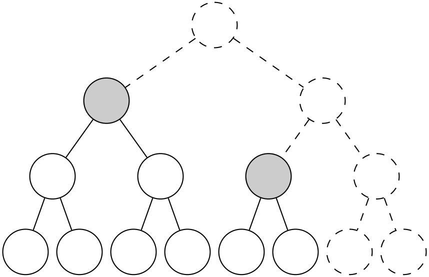

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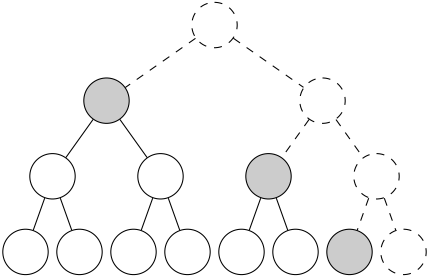

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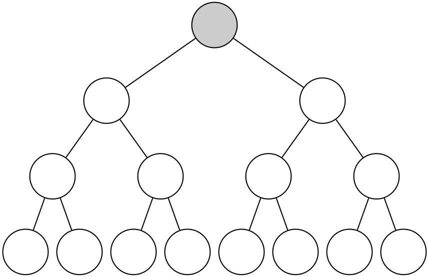

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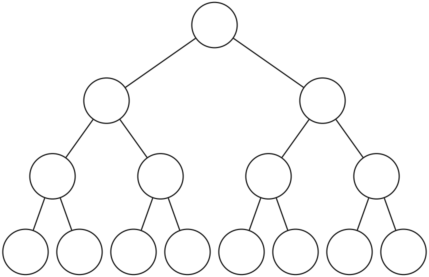

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- Cannot store expanded key material
- Interleave ChaCha12 and Treehash

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- On 4-core Haswell:

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- Hash-based? ChaCha cycles account for nearly 70%!

TODO

- Implement verification
- Implement ChaCha in ARMv7-M `asm`
- Operate on messages of arbitrary size
- Cache (partial) authentication paths

Conclusions

- SPHINCS could replace RSA / ECC / ... for signing
  - Stateless $\rightarrow$ drop-in replacement
  - Conservative security choice
- Feasible on limited platforms
  - Hard memory limit: ✓
  - Time efficiency: gradual optimisation

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- Signature on N-bit value, e.g. 1010 0110 0101 1100
  - For this example, assume $m = 4$, so $w = 16$

$h^{10}(s_0) \;\; h^6(s_1) \;\; h^5(s_2) \; h^{12}(s_3)$

The Winternitz improvement

- Trade time for signature and public key size
- Idea: sign groups of m bits, let $w = 2^m$
- Private key: N/m random numbers

s_0 s_1 s_2 $\cdots$ $s_{N/m-3}$ $s_{N/m-2}$ $s_{N/m-1}$

- Public key: hash w times

$h^w(s_0)$ $h^w(s_1)$ $h^w(s_2)$ $\cdots$ $h^w(s_{N/m-3})$ $h^w(s_{N/m-2})$ $h^w(s_{N/m-1})$

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$h^{10}(s_0)$ $h^6(s_1)$ $h^5(s_2)$ $h^{12}(s_3)$

- Verification: complete hashes to w, check with public key

HORST

- Few-time signature scheme, two parameters k, t, (e.g. $k = 32$, $t = 2^{16}$)
- Private key: t random numbers $s_0, s_1, \ldots, s_{t-1}$
- Public key: $h(s_0), h(s_1), \ldots, h(s_{t-1})$

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  - Build a Merkle tree on top
- Signature on N-bit value (e.g. $N = 512$)
  - Split message (digest!) into k parts
  - Interpret message parts as integers $m_0, m_1, \ldots, m_{k-1}$
  - Reveal $s_{m_0}, s_{m_1}, \ldots, s_{m_{k-1}}$
  - Include authentication paths

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  - Include authentication paths
- Very small chance of re-use